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Inaccuracy of Nozzle Performance Predictions Resulting from the Use of an Invalid Drag Law

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ESSENTIAL to the analysis of gas-particle flow in a rocket nozzle and prediction of performance inefficiency due to the particles' thermal and velocity lags is information on the particle drag coefficient. Studies have shown[1,2] that the flow region encountered by a particle in a rocket nozzle lies within Reynolds numbers less than 10 and Mach numbers less than unity and encompasses the entire gamut of flow regimes from continuum to free-molecule flow. Until now no information has been available for the particle drag coefficient in this region and several different drag laws have been devised and utilized for nozzle-flow analyses and performance predictions. Drag coefficient data have now become available,[3] however, and the adequacy of the various drag laws can be assessed. This Note illustrates the errors in nozzle performance predictions which result from using the devised drag laws and shows the significance of the drag law in nozzle performance predictions.

A survey by Miller and Barrington[4] on the performance prediction of solid propellant rocket motors revealed that four different expressions for the particle drag coefficient have been used in the analyses of gas-particle flows in rocket nozzles. In the earlier works[5] the classical drag coefficient for a sphere in incompressible flow was employed. The discovery that the particles were not in continuum flow for the greater part of their trajectory through the nozzle led to the invention and use of three different drag laws designed to account for rarefaction effects. Kliegel[6] employed a theoretical equation which derives from a solution of the Thirteen Moment equations, namely,

$$C_D = C_{D_0}\left[\frac{(1+7.5Kn)(1+2Kn)+1.91Kn^2}{(1+7.5Kn)(1+3Kn)+(2.29+5.16Kn)Kn^2}\right] \quad (1)$$

where C_{D_0} is the drag coefficient for a sphere in incompressible flow and Kn is the Knudsen number. Carlson and Hoglund,[2] combining rarefaction effects measured by Millikan[7] and Reynolds number trends in incompressible flow, developed the following expression

$$C_D = \frac{24}{Re}\times \left[\frac{(1+0.15Re^{.687})\{1+\exp[(0.427/M^{4.63})-(3/Re^{0.88})]\}}{1+(M/Re)[3.82+1.28\exp(-1.25\,Re/M)]}\right] \quad (2)$$

where Re and M are the particle Reynolds number and Mach number, respectively. Fitting available data at high and low Mach numbers with reasonable trends between, Crowe[8] devised the following drag law

$$C_D = (C_{D_0}-2)\exp\left[-3.07\gamma^{1/2}\frac{M}{Re}g(Re)\right] + \frac{h(M)}{\gamma^{1/2}M}\exp\left(\frac{-Re}{2M}\right)+2 \quad (3)$$

where

$$\log_{10} g(Re) = 1.25[1+\tanh(0.77\log_{10} Re - 1.92)]$$

and

$$h(M) = [2.3+1.7(T_p/T_g)^{1/2}] - 2.3\tanh(1.17\log_{10} M)$$

with T_p and T_g being the particle and gas temperatures, respectively. Because of the absence of drag data in flow regimes encountered in a rocket nozzle, it was not possible to decide which of the preceding laws was most valid.

The problem of the particle drag coefficient in a rocket nozzle has now been resolved by data[3] taken with a microballistic range. These data were reduced in terms of a normalized coefficient suggested by Sherman,[9]

$$\bar{C}_D = (C_D - C_{D_I})/(C_{D_{FM}} - C_{D_I}) \quad (4)$$

where C_{D_I} is referred to as the "inviscid" drag coefficient, taken from high Reynolds-number data for spheres, and $C_{D_{FM}}$ is the drag coefficient for a sphere in free molecule flow.[10] The data from the microballistic range, together with other published data, were plotted vs Knudsen number and an empirical equation was developed to fit the data.

The adequacy of the devised drag laws to represent the data[3] is illustrated in Fig. 1. The expressions used by Kliegel and Carlson represent the data reasonably well at the smaller Knudsen numbers but diverge significantly from the data in the near- and free-molecule flow regimes. The expression proposed by Crowe shows better agreement but exhibits an inflection which is neither observed in the data nor predicted by theory.[11] Based on the data in Fig. 1 and other published data extending down to a Knudsen number of 10^{-3}, the fol-

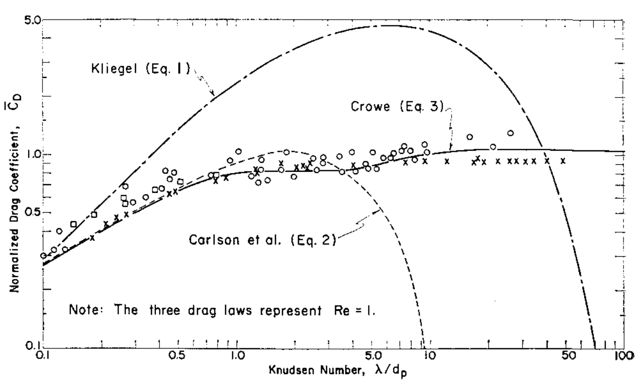

Fig. 1 Adequacy of the devised drag laws to represent data for the particle drag coefficient in flow regimes encountered in a rocket nozzle.

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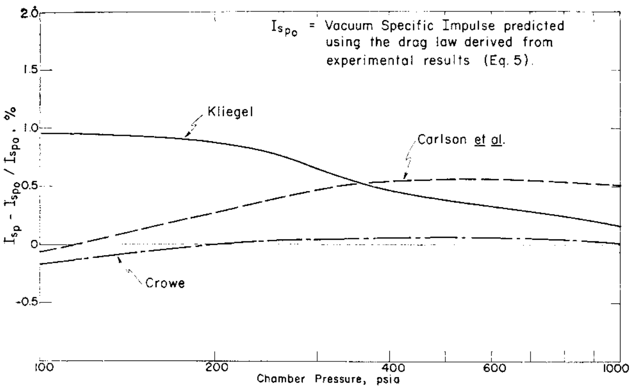

Fig. 2 Percentage error in vacuum specific impulse resulting from the use of the devised drag laws.

lowing empirical expression was developed to best fit the data,

$$\bar{C}_D = \frac{1.1}{1 + 1.1Kn^{-0.3}\exp(-Kn^{1/2})} \times \left\{1 - \exp\left[-Kn^{0.7}e^{Kn}\frac{(C_{D0} - 0.48)Re}{6.6}\right]\right\} \quad (5)$$

The purpose of the Note is to demonstrate the effect on nozzle performance predictions of using the drag laws devised before data were available. The calculations were performed using a computer program[1] for one-dimensional two-phase flow with chamber conditions typical of a solid-propellant motor and chamber pressures ranging between 100 and 1000 psia. Mass median particle sizes as measured by tank collection tests[1] from subscale motors were used in the calculations at the various pressure levels. Vacuum specific impulses of a nozzle with a 40:1 expansion ratio and a 1.2-in. diam. throat were calculated employing the various drag laws.

The percentage difference in nozzle performance predictions resulting from using each devised drag law and that validated by experiment [Eq. (5)] is illustrated in Fig. 2. At low-pressure levels the drag law used by Kliegel leads to a performance prediction which is 1% too high whereas the use of the other two drag laws results in very small differences in performance prediction. At the high-pressure levels, where rarefaction effects are less significant, Carlson's law results in the least accurate prediction although still less than 0.5%. Performing the same calculations for larger nozzles shows that the choice of drag law has even less effect on performance predictions because of reduced two-phase flow losses with increased nozzle scale.

Using the drag coefficient for a sphere in incompressible flow as the drag law in the flow analysis of the same nozzle considered previously results in a vacuum specific impulse which is 3.7% too high at a chamber pressure of 100 psia. An inaccuracy of this magnitude would lead to an intolerable 10–12 point error in the specific impulse prediction of a high-performance low-pressure motor to operate in a space environment, thereby showing the importance of accounting for flow-rarefaction effects.

In conclusion, the drag laws devised before data were available to account for rarefaction effects have not led to large errors in specific impulse predictions. Thus discrepancies between predicted and measured nozzle performance which, in the past, were attributed to lack of knowledge of the particle drag coefficient were likely a result of the still unresolved problem of condensed-phase particle size.

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Modification of Radiant Enclosure Equations for Third Generation Digital Computers

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Nomenclature

$NTEMP$	= number of surfaces with specified temperature
$NFLUX$	= number of surfaces with specified heat flux
NS	= $NFLUX + NTEMP$ = total number of surfaces in enclosure
A_i	= surface area i
F_{ij}	= geometric view factor from surface i to j
B_i	= radiosity or total flux leaving surface i
T_i	= temperature of surface i
ϵ_i	= emissivity of surface i
Q_i	= heat flux to surface i
δ_{ij}	= Kronecker delta = 0 if $i \neq j$; 1 if $i = j$
σ	= Stefan-Boltzmann constant
ψ(INDEX)	= element of symmetric-matrix inverse
INDEX	= equivalent one-dimensional subscript replacing i and j

Introduction

Modern literature on radiation heat transfer derives and uses the equations of radiant interchange in matrix form for solution on digital computers. These equations generally are not written to use minimum core storage. Yet, two compelling reasons dictate that this requirement be met. One is

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